MATH 205: FOUNDATIONS OF ADVANCED MATHEMATICS – FALL 2026

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Class Meetings:	MWF 11am–11:50am, MECC 260
Office hours:	TBD

Course description

This course serves as the starting point for the mathematics major. It is intended as a friendly environment for you to learn about what *mathematics*—that is, the sort of thing mathematicians like me spend our time thinking about—is all about. You might find this course quite different from mathematics you've encountered before: instead of focusing primarily on calculations, this course will be structured around definitions, theorems, lemmas, and of course, proofs.

The central goal of the course is to teach you how to read, understand, and write mathematical proofs. You'll learn how to formulate and manipulate mathematical statements, how to recognise the logical structure of an argument, and how to use the major techniques of mathematical proof. By the end of the course, you should be able to:

- Formulate mathematical statements precisely and analyse their logical structure.
- Use definitions accurately and apply them to prove elementary mathematical statements.
- Construct proofs using direct proof, proof by contrapositive, proof by contradiction, the well-ordering principle, and mathematical induction.
- Read and analyse mathematical proofs, identifying their structure, key ideas, and proof techniques.
- Communicate mathematical arguments clearly, precisely, and rigorously, both orally and in writing.

We won't learn proof-writing as an isolated concept. Instead, we'll develop these skills by rigorously exploring some of the most fundamental—and beautiful—topics in mathematics.

Unit 1 (2–3 weeks): We'll begin with a rapid overview of propositional logic, which will give us the language and logical tools we need to formulate mathematical statements precisely and understand the major methods of proof.

Unit 2 (5–6 weeks): We'll then put these tools to work in elementary number theory, studying divisibility, prime numbers, and rational numbers. Highlights in this unit include some of the greatest hits in mathematics history: Euclid's proof that there are infinitely many primes, the Pythagoreans' discovery that $\sqrt{2}$ is irrational, and the *Fundamental Theorem of Arithmetic*, the fact that every positive integer can be factorised *uniquely* as a product of prime numbers.

Unit 3 (4–5 weeks): Finally, we'll turn to elementary set theory, studying sets, functions, relations, and cardinality. In this unit, we'll give "counting" a precise mathematical definition and push this idea to its limits before exploring infinite sets, culminating in Cantor's proof that some infinities are bigger than others. If time permits, we'll conclude by exploring some of the surprising directions these ideas lead, including Russell's paradox, the Halting problem, and Gödel's incompleteness theorem.

Course materials

There are no required texts for this course. However, you may find the following textbooks helpful:

- *How To Prove It: A Structured Approach*, Daniel J. Velleman.
- *Book of Proof*, Richard Hammack.

Course assignments

I will typically assign two written homework assignments per week. These assignments will contain two types of questions:

Concept check questions. These are short questions designed to help you check that you have understood the basic ideas from class, for example, that you understand a new definition, can interpret a mathematical statement correctly, or can carry out a simple proof.

Concept check questions will not be graded. I will provide solutions so that you can check your understanding for yourself. You should use these questions as a diagnostic: if you find them difficult, that is a good indication that you should review the material or come to office hours before moving on to the graded problems.

Graded problems. These problems are designed to develop your ability to solve mathematical problems and, especially, to construct and communicate rigorous proofs. They are intended to be challenging. It is completely normal to try an idea, discover that it does not work, and have to start again. Learning to write proofs is a process, and your first attempt at a problem does not have to be your last.

Each genuine attempt at a graded problem will be assigned one of four levels: **Mastery (M)**, **Partial Mastery (PM)**, **Progressing (P)**, or **Insufficient (I)**.

If your original submission earns **PM**, **P**, or **I**, you may make exactly **one revised attempt**, following the instructions in the table below. Your revised attempt will be graded independently, using the same four levels and the same standards as your original submission. Your recorded score for the problem will be the higher of the two scores.

If you do not make a meaningful attempt at a graded problem, you will receive a score of 0 for that problem and will not be eligible to submit a revised attempt.

Your homework grade will be the average of the recorded scores on your graded problems. Concept check questions do not contribute to your grade.

Level	Score	What it means	What happens next
M	100	Your solution demonstrates mastery of the problem: • the mathematical argument is correct and complete; • the proof is written clearly, precisely, and with good mathematical style; • a few minor errors or imprecisions are acceptable, provided that they do not detract from the validity, clarity, or overall quality of the solution.	You're done!
PM	85	Your solution is essentially correct and demonstrates the main ideas, but falls short of full mastery in one or more ways: • there is an important but readily fixable error, gap, or imprecision; • the mathematical writing is not yet sufficiently clear or precise for mastery; • there are too many minor errors or imprecisions for the solution to demonstrate full mastery. The argument can be corrected without substantially changing its overall approach.	Revise and resubmit the same solution.
P	70	Your solution demonstrates meaningful progress toward a correct proof, but does not yet demonstrate partial mastery: • there is a substantial error or gap; • an important idea is missing or not used correctly; • the argument requires a significant change before it becomes a complete proof; • the mathematical writing is not yet sufficiently clear or precise for partial mastery. The argument cannot be corrected without substantially changing its overall approach.	Solve and submit a new, similar problem.
I	55	Your solution does not yet demonstrate sufficient understanding of the central idea or skill required by the problem: • the main approach is missing or substantially incorrect; • the solution makes insufficient progress toward a valid proof.	Solve and submit a new, similar problem.

Mathematical writing. Your solutions will be assessed not only for mathematical correctness but also for the quality of your mathematical communication. A proof is an argument written for another person to read: it should make clear what you are claiming, why each step is justified, and how the steps fit together.

As the semester progresses, we will spend considerable time discussing what good mathematical writing looks like. Your solutions should be written clearly and legibly, in complete sentences where appropriate, and with careful use of mathematical notation, grammar, and punctuation.

Collaboration. You are welcome—and encouraged—to discuss problems and ideas with other students. However, the solution you submit must be your own work, written in your own words. In particular, you should be able to explain and defend every step of any proof that you submit.

Late homework policy. You have three grace days for graded problems and resubmissions during the semester. A grace day lets you turn in a homework assignment up to 24 hours late for any reason, with no explanation needed. You may use these days one at a time or all at once, but once they are gone, no further late work will be accepted without prior approval.

Beyond these three grace days, late homework will only be accepted if you have a valid excuse (e.g. illness, family emergency) that you communicate to me *before the deadline*, and I approve it. Excused late work of this kind does not count against your grace days.

Assessment

Exams. There will be two in-class midterm exams and a comprehensive final exam:

- First midterm: Friday, October 16
- Second midterm: Monday, November 23
- Final exam: Monday, December 21, 12pm–2:30pm

Grading. 90% of your grade will be based on the following distribution:

Homework	30%
Midterm 1	15%
Midterm 2	15%
Final exam	30%

The remaining 10% will be allocated to the highest of your three exam scores.

Grading scale. You will be assigned a letter grade based on the following scale:

A	A-	B+	B	B-	C+	C	C-	D	F
[93,100]	[90, 93)	[87, 90)	[83, 87)	[80, 83)	[77,80)	[73,77)	[70,73)	[60,70)	[0,60)

In accordance with the policy of the Dean of the Faculty, the grade of A+ is reserved for students who truly excel in the course, and would rate as in the top 1% of Trinity students.

Course policies

Attendance and participation. Unless otherwise announced, all class meetings will occur in person, and you are expected to attend each class. *Regular attendance is vital to success in this course,* and repeated absences will impact your ability to keep up with the course material. While you will not be explicitly graded on attendance, in borderline cases, I may take attendance and participation into account when determining final grades.

If you need to miss a class due to illness, college event, or any other reason, please email ariel.weiss@trincoll.edu to inform me, and I'll do my best to help prevent you from falling behind.

Office hours. It is very normal for students to attend my office hours, and I encourage you to do so. Possible reasons you might visit include:

- You are stuck on a homework problem and want to discuss ideas.
- You want to go over an exam or homework problem to get extra feedback.
- You are interested in a concept from class and want to learn more about it.
- You are excited about something mathematical and want to share it with me.

Please be aware that my office hours are generally not private. If you need to speak with me privately, let me know and I will arrange a separate time for us to meet.

Use of Generative AI. As machine learning tools like ChatGPT become more widespread, it's important to use them thoughtfully and responsibly. The course policy on these tools can be succinctly summarised as follows:

- You are *encouraged* to use generative AI tools in ways that enhance your learning.
- You should not use generative AI tools in ways that detract from your learning.
- Submitting AI-generated content as your own work is plagiarism and is strictly prohibited.

Examples of uses of AI that are beneficial include generating lists of practice problems, asking for explanations of a concept that you are struggling with, having AI quiz you on your understanding, asking for clarification of unfamiliar terminology or reviewing definitions, using AI to help brainstorm questions to ask during office hours or study group sessions, and requesting a worked example of a problem type *after* attempting similar problems yourself.

Note that AI tools may provide information that is wildly incorrect or needlessly complicated, and that using AI is only beneficial if you are sufficiently sceptical of its output.

Examples of uses of AI that are not beneficial include asking for hints for current homework problems, requesting a worked example of a type of problem *before* you have attempted similar problems yourself, asking AI to improve your mathematical writing, and using generative AI in place of collaborating with your peers.

Examples of uses of AI that reach level of plagiarism include any use of AI that results in you submitting AI-generated material, even if you substantially revise or reword it. This includes copying explanations, computations, or proofs, regardless of how much you edit them.

These categories are necessarily subjective, with many grey areas. If you are in any doubt whether a use case of AI is beneficial, not beneficial, or prohibited, please err on the side of caution and ask me first!

Academic Integrity. Academic honesty is highly valued at Trinity. In accordance with the Trinity College Student Integrity Contract, students are expected to abide by the highest standards of intellectual honesty in all academic exercises. Intellectual honesty assumes that students do their own work and that they credit properly those upon whose work and thought they draw. It is the

responsibility of each student to make sure that they are fully aware of what constitutes intellectually honest work in every examination, quiz, paper, laboratory report, homework assignment, or other academic exercise submitted for evaluation in a course at Trinity College.

Academic Accommodations. Trinity College is committed to creating an inclusive and accessible learning environment consistent with the Americans with Disabilities Act. Students with disabilities who may need some accommodation in order to fully participate in this class are urged to contact the Student Accessibility Resource Center, as soon as possible, to explore what arrangements need to be made to assure access.

If you have approval for academic accommodations, please notify me by the end of week two of classes. For those students with accommodations approved after the start of the semester, a minimum of 10 days' notice is required. Please be sure to meet with me privately to discuss implementation.

Student Accessibility Resources can be reached by emailing SARC@trincoll.edu.

Disclaimer. This course syllabus is a general plan for the course, however, deviations may be necessary. If I need to change a course policy, I will make an announcement to the class and give sufficient notice.