Math 228-01 Linear Algebra
Fall 2024

Professor: M. Sandoval, Nutt MECC 219, 297-2062, email: msandova@trincoll.edu (The best way to contact me!)

Time and Location: MWF 10:00 - 10:50pm, Location: Nutt MECC 260.

Couse Website: We have a Moodle Website in the usual place.

Office Hours: (See Moodle site for the most up-to-date schedule of office hours): TBA

Linear algebra is rich subject which has deep connections to many areas of mathematics and statistics. It is also incredibly useful with applications ranging from engineering and aircraft design, special relativity in physics, understanding electrical networks, balancing chemical equations, computer graphics, econometrics, statistics, and population dynamics. It is also deeply useful in understanding abstract mathematics, which is our focus. You will recognize many ideas from other courses such as differential equations, as well as be introduced to ideas that will appear in your later coursework as mathematics majors, including group theory and abstract algebra. Many students who have taken this course have returned to me years later to recall this course as their favorite mathematics course.

This course is serves three purposes: (1) it provides a complete exposition of the theory of linear equations, and their solution sets, as well as the structure of these solution sets; (2) it introduces abstract algebraic structures, and their symmetries, necessary to the understanding of higher mathematics; (3) it is designed to help students develop their proof-writing skills, which are essential for mathematics majors. In light of this third purpose, be aware that students who choose not to develop their proof-writing skills will not receive a grade higher than a D in this course. However, if you are interested in developing this skill, you will have opportunities to practice these skills and develop them further via weekly re-writes of proofs on the homework sets.

By the end of this course, successful students will be able to:

1. Given a linear system of equations, students will be able to identify what type of solution set to expect (by applying the theory of linear systems), and be able to compute it, or find its best-fit approximate solution when it does not have an exact solution.

2. Students should be have a working knowledge of the algebraic rules for various classes of mathematical objects (such as matrices, real numbers, abstract vectors, inner product spaces), compare and contrast them, and apply these rules to simplify various expressions.

3. Students will be able to closely read definitions, arguments, identify common structures, apply abstract criteria to determine if particular examples meet the abstract hypotheses or criteria, and be able to explain the logical relationship between theorems and definitions in this course.

4. Students will be able to closely read proofs, identify logical structures, and identify commonly used techniques.

5. Students will be able to reflect on an argument, assess its validity, analyze its structure, comparing it to similarly structured arguments, and revise it as needed.

6. Students will be able to construct correct, concise arguments (proofs), making well-justified rigorous statements, sometimes from first principles, and write them up clearly, using good style, exposition, and form.

More specifically, in terms of subject matter, successful students will be able to

- Determine when a system of linear equations has a unique solution, and infinite solution set, or an empty solution set.
- Determine the structure of the solution set when it is infinite, using abstract concepts such as vector subspaces.
- Find best-fit solutions when the solution set is empty; this is known as mulit-linear regression.
- Appreciate the utility of matrix methods in solving linear systems.
- Reason abstractly about matrix inverses, determinants, and matrix algebra.
- Understand and be able to reason abstractly with concepts such as subspaces, dimension, and bases.
- Generalize matrices to linear transformations and be able to reason abstractly with them.
- Understand and reason with abstract definitions of distance in higher dimensional spaces.
- Understand, and be able to compute, and abstractly reason with eigenvalues and eigenvectors of linear transformations.

Office Hours: It is quite normal for students to regularly make use of my office hours. In office hours, I will help you learn how to do the homework on your own. I will not do any of the assigned homework problems *for* you. I will gladly review similar problems as examples for you to follow, or review general techniques and strategies. It is your responsibility to see me if you are having difficulty with the homework in a timely manner. Waiting until the last minute is too late. I will distribute solutions to the computational homework once it has been turned in. Please note: you should start the homework soon enough so that you can take advantage of my office hours. If you wait until the last minute to begin the homework, you will not be able to make use of office hours and you will find yourself forced into turning in an incomplete or badly done assignment. I have carefully scheduled my office hours and the due dates of homework assignments so that you should have ample opportunities to see me about difficulties that arise.

Due to COVID19 protocols, office hours *may* be held via Zoom, depending on current conditions. I will post a link to my Zoom office hours on the course website on moodle.

Textbook: *Linear Algebra and Applications,* Third Edition, by R. Hill. This text is available for free as a pdf on our moodle site. You will need to read the relevant sections of the textbook (see below). In fact, it will often be necessary for you to read the text and examples to do the homework, as we will not have time in class to explore every wrinkle in the material that might come up in the course of a homework assignment. It should also be emphasized that the text is just one resource used in this course. It is designed to complement the material discussed in class, not replace it. The content of this course is more than just what appears in the text. Attendance and participation in the course includes that material discussed in class in addition to the material that appears in the text.

Material: Chapters 1 through 5, most sections.

Grading: This course is primarily about learning how to prove mathematical facts, rather than computations. Problems involving mathematical reasoning will be weighted more than problems

involving computations on all tests and homeworks. The course grade will be based on the following distribution:

Videos & Online Concept Check Quizzes	10%
Attendance/Quizzes/In-Class Activities	15%
Weekly Proof/Computational Sets	40%
Exam I	
Exam II	20 % (total for both exams)
Final on Wednesday, December 18th, noon	15 %

Extra credit beyond the above is not available.

The letter grade in this course is based on a straight scale, with the exceptions (1) that I do not decorate D grades with plusses or minuses; and (2) the grade of A+ is reserved for the truly remarkable students–someone whom I would rate in the top 1 percent of all the students I have encountered in my career at Trinity (this is consistent with the policy of the Dean of the Faculty's latest policy)–consequently, a grade that is a 90 percent but below a 95 percent is an A- and a grade above 95 percent is an A.

Attendance and Classroom Deportment: Successful students attend class. Attendance will be taken and will be taken into account in the calculation of the final grade. To receive full credit attendance you must attend class for the full class period **and be engaged in class activities**. We will be using a work-shop model to actively work on the assignments during a portion of the class period.

In-Class Worksheets/Quizzes/Class Activities: We will have regular in-class activities in the form of worksheets, quizzes, or other class activities. Some of the quizzes may be posted before class for you to complete on moodle. If you are absent, you may not make up the missed worksheet or quiz except under extraordinary circumstances that can be documented via the Dean of Students Office.

Proof-Writing Homework Sets: These are the main focus of the course. Problem Sets will usually consist of 3-5 problems. All assignments must be turned in. Assignments are due at the beginning of class on the day announced on the assignment. Electronic submission as a single pdf are preferred. Late homework will receive a zero, and will not be graded. Electronic submissions of your proof-writing homework as a single pdf is encouraged. It does not need to be typeset with a program; hand-written work is fine as long as it is organized and legible.

Rewrite policy: Learning a new skill like proof-writing requires practice and revision. In order to help you learn this skill I will allow you to submit revisions of proofs as follows. You may rewrite any problem up to a 80 percent. The revisions must be turned in within one week of the day that the proof was returned to class, whether you were present in class that day or not. You may not rewrite a skipped problem or a problem turned in late, assuming it is accepted. When preparing homework leave wide 1 inch margins all around and 2 inches between problems, to allow for space for comments to be written.

Exams: There will be two exams. There will be no make-up exams. If you suffer from a sudden illness or emergency that prevents you from taking an exam, then you must notify me and the Dean of Students by email, telephone message, or other means of communication prior to or during that exam. If your emergency is deemed sufficient to excuse you from the exam, you will not be given a make-up exam–your final exam will simply be lengthened and rescaled to make up for the lost points. A missed exam that has not been excused will result in a zero score. The dates of the two exams are as follows:

Wednesday, October 2nd
Wednesday, November 6th

The Final: The final exam is scheduled for Wednesday, December 18th, at noon.

Homework Submissions: Homework Assignments will be submitted through our moodle site.
Attendance and Classroom Deportment: Attendance **and class participation** are a portion of your grade (see above). If you are working on the assigned tasks in class you will be awarded full credit for attendance and class participation, and will be taken into account in the calculation of the final grade. If a pattern of non-attendance or non-participation becomes apparent, midterm grade reports make note of this pattern. The use of cell phones are not permitted during class for any reason, but laptops and tablets may be used for consulting videos/submitting assignments, and working on class activities, provided that they do not become a distraction to other students. If you have questions in class, please ask them, verbally. In the classroom, there are many things that have my attention, and if you just raise your hand without saying anything, I may not see your hand. It is my expectation that you will verbally get my attention if you should need it.

Academic Honesty: Academic honesty is highly valued at Trinity. In accordance with the Trinity College Student Integrity Contract, students are expected to abide by the highest standards of intellectual honesty in all academic exercises. Intellectual honesty assumes that students do their own work and that they credit properly those upon whose work and thought they draw. In this course, you may not draw upon internet solutions or the work of others. It is the responsibility of each student to make sure that he or she is fully aware of what constitutes intellectually honest work in every examination, quiz, paper, laboratory report, homework assignment, or other academic exercise submitted for evaluation in a course at Trinity College.

Academic Honesty and Use of AI in this course: Academic Honesty is highly valued at Trinity College and is taken very seriously. The use of an AI (Artificial Intelligence) generator such as ChatGPT, iA Writer, MidJourney, DALL-E, et cetera is explicitly prohibited unless otherwise noted by Professor Sandoval. The information derived from these tools is based on previously published materials. Therefore, using these tools without citing the underlying source material constitutes plagiarism. Additionally, you should be aware that the information derived from these tools is often inaccurate and/or incomplete. It is imperative that all work submitted should be your own work, your own ideas, based on your own reflections or experiences. Any assignment that is found to have been plagiarized or to have used unauthorized AI tools may receive a zero grade and be reported for academic misconduct to the appropriate disciplinary committee.

Students with Academic Accommodations: Trinity College is committed to creating an inclusive and accessible learning environment consistent with the Americans with Disabilities Act. Students with disabilities who may need some accommodation in order to fully participate in this class are urged to contact the Student Accessibility Resource Center, as soon as possible, to explore what arrangements need to be made to assure access.

If you have approval for academic accommodations, please notify me by the end of week two of classes. For those students with accommodations approved after the start of the semester, a minimum of 10 days' notice is required. Please be sure to meet with me privately to discuss implementation.

Student Accessibility Resources can be reached by emailing SARC@trincoll.edu.

Finally: Unless stated otherwise, all other matters of course policy are subject to the instructor's discretion and may be changed with suitable notice.

Tentative schedule: (for a current version, look at the moodle schedule)

1	W	Sep 4	1	Topic 1: Introduction to Linear Algebra
1	F	Sep 6	2	Topic 2: Gaussian Elimnation
1	M	Sep 9	3	Topic 3: The Algebra of Matrices
2	W	Sep 11	4	Topic 4: Matrix Multiplication, HW Set 1 due
2	F	Sep 13	5	Topic 5: Multiplication as Composition
2	M	Sep 16	6	Topic 6: Multiplicative Inverses
3	W	Sep 18	7	Topic 7: Invertibility/Row Equiv., HW Set 2 due
3	F	Sep 20	8	Topic 8: Invertibility Criteria
3	M	Sep 23	9	Topic 9: The Matrix Transpose
4	W	Sep 25	10	Topic 10: The Determinant Function, HW Set 3 due
4	F	Sep 27	11	Topic 11 Properties of Determinants, part 1
4	M	Sep 30	12	Topic 12: Properties of Determinants, part 2
5	W	Oct 2	13	**Exam 1 on Topics 1--9, HW Set 4 due**
5	F	Oct 4	14	Topic 14: Methods of Calculating Determinants
5	M	Oct 7	15	Topic 15: Sets of Vectors, Spans
6	W	Oct 9	16	Topic 16: Gram-Schmidt Orthogonalization, HW Set 5 due
6	F	Oct 11	17	Topic 17: Vector Spaces
	M	Oct 14		Trinity Day--No classes
6	W	Oct 16	18	Topic 18: Vector Subspaces
7	F	Oct 18	19	Topic 19: Linear Independence
7	M	Oct 21	20	Topic 20: Linear Independence/Dependence

7	W	Oct 23	21	Basis and Dimension
8	F	Oct 25	22	Fundamental Subspaces of a Matrix
8	M	Oct 28	24	Coordinates with respect to a Basis
8	W	Oct 30	25	Linear Transformation/Inner Product Spaces
9	F	Nov 1	26	Important Inequalities
9	M	Nov 4	27	Orthogonal Projection/Orthogonal Matrices
9	W	Nov 6		**Exam 2**
10	F	Nov 8	28	Invariant Subspaces of Linear Transformations
10	M	Nov 11	29	Eigenvalues/Eigenvectors
10	W	Nov 13	30	Diagonalization of Matrices
11	F	Nov 15	31	Properties of Diagonalizable Matrices
11	M	Nov 18	32	Symmetric Matrices & Diagonalizability
11	W	Nov 20	33	Complex Matrices and Eigenvalues
12	F	Nov 22	34	Diagonalization of Complex Matrices
12	M	Nov 25	35	The Spectral Theorem
	W	Nov 27		Fall Break--No classes
	F	Nov 29		Fall Break--No classes
12	M	Dec 2	36	The Proof of the Spectral Theorem
13	W	Dec 4	37	Least Squares Problem
13	F	Dec 6	38	Application: Multilinear Regression
13	M	Dec 9	39	Review for the Final Exam
	W	Dec 18	40	Final Exam

Is Linear Algebra right for you?

Linear Algebra may not be right for you at this time if any of the following are true:

- If you have really struggled in the past with proof and writing proofs and don't have a compelling motivation to overcome this discomfort.
- If you have a history of not taking advantage of office hours.
- If you are taking a heavy course load and may not have enough time to put into this course.
- If you have a tendency to miss deadlines or regard opportunities to submit rewrites as "optional" assignments.
- If you don't like "theory"– only applications.

On the other hand, you should consider taking this course if any of the following apply to you:

- If you really want to learn how to write proofs.
- If you think you might want to be a mathematics major.
- If you want to understand how different subjects in mathematics are connected.
- If you feel as though you really want to deeply understand a mathematical subject.